The Mysterious Constant Alpha (α) in Quantumphysics

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Abstract The question of how to calculate the numerical value of the fine structure constant from theory was one of the most outstandingly difficult problems in mathematical physics for the greater part of the 20-th century. There were many unsuccessful attempts by researchers including famous ones such as Dirac to find a formula for the fine structure constant. This study is an attempt to demonstrate how to find a mathematical description and to find an idea of the enormous effects this dimensionless constant has to physics. The formula was found due to the incorporating of the maximal speed a particle can reach. Since this speed is slightly below speed of light but not even relativistic treatment of atoms and other structure must involve corrections that are in its simplest form given by the mysterious factor \( \alpha \) called the fine-structure constant. It is shown that the fine-structure constant is a direct consequence of the limited speed of an electrical charged particle in vacuum.

Keywords: special relativity, classical field theory, quantum mechanics


1. Introduction

The fine-structure constant is a dimensionless physical constant presenting a value for the electromagnetic interaction and plays an important role in classical theories as well as in modern quantum mechanics and QED. It seems to be a kind of key link from the nineteenth century until present including philosophical mysteries (see for an overview. The fine structure constant \( \alpha \), strictly speaking only emerged as a specific entity out of Sommerfeld’s theory for the fine spectroscopy structure of the hydrogen atomic states [1]. The birth of the constant occurred in this paper and presents a formula for the Hydrogen spectrum in quantum numbers \( n \), \( l \) to expand the energy term in the Bohr’s atom model

\[
E_{ FS} = \frac{E_n}{n} \left[ \frac{1}{f + \frac{1}{2}} - \frac{3}{4N} \right] Z^2 + \ldots
\]

with the physical definition

\[
\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \quad \text{or} \quad \alpha = \frac{1}{2\epsilon_0 \hbar c},
\]

and this is \( \alpha \approx 1/137 \). The \( e^2 / \hbar \) is the reciprocal of the von Klitzing’s elementary resistance, which can be determined from the Quantum-Hall effect. The fine-structure constant has several interpretations and some are mentioned here:

1. the square ratio of the elementary charge to the Planck charge
2. the ratio of two energies: (a) the energy needed to overcome the electrostatic repulsion between two electrons a distance \( d \) apart

\[
E = \frac{e^2}{4\pi\epsilon_0 \hbar c / \lambda} = \frac{h^2}{4\pi\epsilon_0 \hbar c \lambda}.
\]

and (b) the energy of a single photon of wavelength \( \lambda = 2\pi d \)
3. the ratio of the velocity of the electron in the first circular orbit of the Bohr atom to the propagation speed of light \( c \) in vacuum [1].
4. the two ratios of three characteristic lengths: the classical electron radius \( r_e \), the Compton wavelength of the electron \( \lambda_C \), and the Bohr radius \( a_0 \)

\[
r_e = \frac{\alpha \cdot \lambda_C}{2\pi} = \alpha^2 \cdot a_0
\]

5. in quantum electrodynamics \( \alpha \) is directly related to the coupling constant determining the strength of the interaction between electrons and protons. However, QED does not predict its value, and the constant must be gained experimentally. In fact, \( \alpha \) is one of the about 20
6. Given two hypothetical point particles each of Planck mass and elementary charge, separated by any distance, then \( \alpha \) is the ratio of their electrostatic repulsive force to their gravitational attractice force.

Since the observed value of Alfa (\( \alpha \)) is associated with the electron mass, and the electron is lower bound for the energy scale because it – and, of course, the “anti-electron” positron – is the lightest charged object with quantum loops contributing to running, the value \( 1/137 \ldots \) is valid at zero energy. Generally, \( \alpha \) is seen to be a constant, though, experimental elementary physics shows values differing from the above by more than a magnitude, \( e.g. \), on the mass of the Z-boson of 91 GeV revealing a factor of 1/128. Here, it comes to the forth the interaction is screened by electron-positron pairs that exist momentarily out of the
vacuum, i.e., vacuum fluctuation. The particles come closer at higher energies and thus there are less electron-positron pairs between them shielding the interaction. The exact value of the constant is defined \[2\].

\[
\alpha = 7.29 \times 10^{-3}
\]

or \(\alpha^{-1} = 137.035999074 (44) \times 10^{-3}\) \(5\)

It still stands the question “how can the fine-structure constant value obtained from theory?” Obviously, there was no theory during many decades and probably the reason so few formulae giving values for \(\alpha\) following Sommerfeld’s work. There was no idea what forms such a theory would take if it is found; there was also no idea what form any formula that might arise would take or what it would depend on.

A first impressing picture follows from Schrödinger’s theory including the Coulomb potential: the speeds for the Bohr orbits with \(Z = 1\) and \(Z = 137\) are

\[
v_1 = 1(\alpha \cdot c) \text{ and } v_{137} = 137(\alpha \cdot c)
\]

The energies of the first Bohr orbits for various values \(Z\) given by \(n = 1, j = 1/2,\) and \(1 \leq Z \leq 137\) is

\[
E_{1,\alpha, Z} = \frac{\mu_0 c^2}{\sqrt{1 - (Z\alpha)^2}}
\]

This formula arising from the Bohr theory \([3,4]\), however, does not take into account relativistic effects (e.g., \([5]\)) and especially the Lamb shift \([6]\) based on the interaction of the electron with the vacuum, and the quantum vacuum \([7,8,9]\). The interaction of the electron with the vacuum, and Planck’s constant thus leading to the fine structure Landé g-factor.

The dimensionless magnetic moment of the electron is referred to as the comparison of the electron charge, therefore Maxwell’s theory and construct a mathematical model relating to its charge

\[
h\alpha = \frac{\text{charge of electron}}{\text{Planck’s constant}}
\]

2. Theory

The current study is based on recent results found on the discussion of superluminal movements of real particles \([13]\). It is based on the Heisenberg uncertainty and the approximation introduced in the former study of superluminality. Here, in a recent study electrostatic attraction between a subluminal bradyon particle \(B\) with its superluminal tachyon co-particle or anti-particle is postulated to possess symmetry with regard to real \(C\)-operation, and \(CPT\)-operation is required for symmetry conservation for anti-particles. As a consequence of the Lorentz transformation the speed \(v\) of the particle observed leads to “deformation” of the electrical field. It has been postulated that under a change from sub- into superluminality the Lorentz transformations change in sign therefore transforming a bradyonic electrical-field vector \(+E_B\) into a tachyonic \(-E_T\) entailing the superluminal co-particle \(T\) to behave being opposite in electrical charge \(q\) to the subluminal particle \(B\) in consequence of Coulomb’s law. The transition from subluminal into superluminal state was postulated to occur in no time, the energy borrowed from the quantum vacuum. The present work is to be seen as an exact investigation of the two speeds forming the speed of light, i.e., propagation of light, the lower being \(v = c - a\) and the faster of \(v = c + a\). In analogy to the conventional Lorentz factor in Special Relativity it is

\[
Y_B^{-1} = +\frac{1}{\sqrt{1 - v_B^2 / c^2}}
\]

\[
Y_B = +\sqrt{1 + v_B^2 / c^2},
\]

for a bradyon.
$Y_T^{-1} = \frac{1}{\sqrt{1 - \frac{v_T^2}{c^2}}}$

$Y_T = \sqrt{1 - \frac{v_T^2}{c^2}}$;

for a tachyon

$v_B = v, \quad 0 \leq v < c$

$v_T = 2c - v, \quad c < v \leq 2c.$

(10)

Here, a speed $v = c$ was strictly excluded. Further, the second postulate in Special Relativity exactly spoken is “the speed of light in vacuum is a constant” that means the vacuum propagation (!) of light and nothing else.

The expressions (10) together with the Heisenberg uncertainty were introduced to describe the subject of light-barrier crossing. The relativistic calculation based on the electrostatic interaction between two electrical point charges with non-vanishing rest mass and no spin. The result was a formula to restrict light-barrier crossing of a real particle of point charge $q$ and non-vanishing rest mass $\mu_0 \neq 0$ to emphasize exactly two speeds leading a particle from a subluminal condition to superluminal:

$v = \frac{\left(2\hbar\epsilon_0c / q^2\right)}{\sqrt{1 + \left(2\hbar\epsilon_0c / q^2\right)^2}} \cdot c.$

(11)

As an example for the elementary charge $q = e$, negative or positive in sign, the fastest vacuum speed $v$ of a $B$ and the slowest $v$ of a $T$ are then

$v_B = 0.99997301784 \cdot c$

and $v_T = 1.00002698215 \cdot c.$

(12)

These are the “critical speeds” where jump-over at the light-barrier can occur. As a consequence, a vacuum light-speed $c$ (propagation of light) is therefore never reached by a real particle.

The graphics show this behavior in Figure 1 for $Y^{-1}$ and Figure 2 for $Y$. It can be seen the traversal between sub- and superluminal states occur at the point $Y. v/c = 1 \pm 0.00002698215$ marked by an arrow on the the abscissa. Due to the symmetry between the subluminal and superluminal systems only a bradyon $B$ will be considered in the following.

According to the “critical” speed (eg. 11) and the Lorentz transformation (eg. 10) it is

$Y^{-1} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{1 + \left(2\hbar\epsilon_0c / q^2\right)^2}$

(13)

$\frac{1}{1 + \left(2\hbar\epsilon_0c / q^2\right)^2} \approx \frac{q^2}{2\hbar\epsilon_0c} = \alpha$

Quod erat demonstrandum.

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**Figure 1.** The factor $Y^{-1}$ as function of the speed $v / c$ for a particle of the elementary charge $e$ in sub- and superluminal conditions. The arrow on the abscissa marks the speed limits touching the light-barrier.
3. Results

The general result is that the fine-structure constant is a direct consequence of the speed limit a particle can reach a vacuum, which is not inconsistent with Einstein’s second postulate. Though, this postulate often reads “the light-speed in vacuum is a constant” instead of “the speed of light in vacuum is a constant”, the latter means the propagation (!) of light necessary to justify the Lorentz’s transformations.

On the basis of the speed-limit of a particle touching the light-barrier the related value of $\gamma = (1 - vB^2/c^2) \equiv \alpha$ could be calculated. The numerical values are $\alpha = 0.007345990197$ and $\alpha^{-1} = 136.1286869820$.

4. Discussion

It has to be highlighted the present theory based on a calculation omitting any spin as would be necessary to get a more precise result in the limit-speed a particle can reach. Moreover, in the theory regarding this speed an approximation was used to resolve the equation for the comparison of mechanics with electrodynamics to introduce the Heisenberg’s uncertainty. As a consequence, the speed must be slightly faster than used and would lead to an exact expression for $\alpha$ so that an equal sign in eq. (12) becomes true. The value recommended by CODATA [14] is $\alpha = 0.0072973525664$ (17).

Principally, this discussion provides an insight into the circumstances that appear in the difficulties and to the discrepancy in atomic models etc. It can be explained how important this factor is, e.g., in atomic physics, just to point out the critics on the incompleteness of the Bohr model [3] so many years ago. That model deals with the problem of the orbit electrons and their, following the theory, a speed faster that light; in their orbits. One of the first modifications suggested by Sommerfeld [1] was the introduction of the fine-structure constant, that results from electrodynamics and relativistic mechanic and leads to a decrease the value in the expression for the atomic energy. Besides those former results the constant plays a central role in QED and, e.g., optical and ESR and NMR spectroscopy (see, e.g., [15, 16]) to provide information about chemical coupling and chemical compound in various environments. Due to an inside view $\alpha$ follows a demand for the interaction between a charged (particle) and the quantum vacuum as demonstrated by Lamb and Retherford [6]. Since the expression for the speed is exclusively determined by the natural constants $e$, $h$, $c$, $h$, and $\varepsilon_o$ the constant $\alpha$ itself is independent on any variables as well. An influence to the pure atomic behavior can result in the factor $Z$ given by the number in the atomic table just revealing the statement of $Z = 137$ as an upper limit.

5. Summary

It has been shown how the fine-structure constant can be proven using the limiting speeds revealed by electrodynamics and relativistic mechanics coupled by the Heisenberg’s uncertainty. Since those speeds are bordering the light-barrier it was taken into account the speed of a real particle, e.g., an electron can never reach
speed of light, though, it can come very close to it. The fine-structure constant constitutes this fact so that it is included in other calculation and allows both interpretations and conclusions on physical phenomena. It has been demonstrated a value of $Z = 137$ could be could a sensible guess for the highest atomic number.

References